Local Spacetime Dynamics, the Einstein-Straus Vacuole and the PIONEER Anomaly: A New Access to these Problems

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The question concerning the extent of the local spacetime has often been raised. At what circumsolar distance the well known Robertson-Walker spacetime of our expanding universe may become a valid approximation? Inside of that distance a local Schwarzschild metric, which permits to explain the Keplerian motions of planets within the frame of general relativity, must be applicable. We briefly analyze the historical answer to that question given by Einstein, Straus and their followers and show that till now this answer is unsatisfactory in many respects.

We revisit the problem of local spacetime geometries in the light of their effects on local photon propagation in view of the radiopropagation phenomena detected with the NASA spaceprobes PIONEER-10/11, waiting for a satisfying answer for several decades now. Comparing radiosignals outgoing from the earth to the probe and ingoing again from the probe to the receiver on earth do show anomalous frequency shifts which presently find no explanation by anomalous non-Newtonian decelerations of these probes. Therefore we study cosmological conditions for the transfer of radiosignals between the earth and these distant probes based on time dependent local spacetime geometries. First we study the cosmological redshift of radiophotons during their propagation to the spaceprobe and show that this shift in fact explains the registered PIONEER phenomenon under the assumption that the full cosmological expansion of the universe also takes place locally. Though yielding the right magnitude, one finds that this assumption leads to a redshift instead of the observed blueshift. We then, however, show that theoretically motivated forms of time dependent local spacetime metrices in fact lead to a blueshift of the needed magnitude. The appropriate local space vacuole is characterized by a Schwarzschild metric of a central mass increasing with cosmic time. Though it is clear that further studies of this effect have to be carried out to give more credit and verification to this hypothetical result, nevertheless more careful hightech radiotracking of freely flying spaceprobes may do a usefull job in confirming a completely unexpected, but cosmologically highly relevant phenomenon.

Key words: Cosmology; General Relativity; Local Systems.

1. Introduction

1.1. The Problem of Local Spacetime Metrics

Since more than sixty years there exists a solution for the general relativistic connection between gravitationally bound systems, such as the solar system, and the freely expanding cosmological space. This solution is known as the Einstein-Straus vacuole (ES vacuole), since it was first studied by Einstein and Straus [1, 2] and their successors [3, 4]. While this result was widely accepted as an adequate solution to the underlying problem, there are both observational, e. g. [5, 6], and theoretical, e. g. [7], hints to fundamental problems related to the Einstein-Straus metric.

Perhaps the most obvious argument against the vacuole concept is the fact that its radius

$$r_{\mathrm{ES}} = \left(\frac{3M}{4\pi\rho_0}\right)^{1/3}, \tag{1}$$

for a present-day cosmic matter density $\rho_{\mathrm{m}} \simeq 10^{-30}$ g/cm^3 and typical solar masses $M \simeq 10^{30}$ kg, turns out to be much too large compared to the mean distance between any of two neighbouring central masses, and the vacuoles created by, say, two neighbouring suns in the milky way inevitably overlap. And since the Einstein field equations are nonlinear, two neighbouring vacuoles may not simply be linearly superposed. Such an overlap also contradicts the initial assumptions made in [1], where the vision occurred that the entire universe might be filled by such discrete (non-overlapping) vacuoles. The present-day view is that smaller objects, such as suns and galaxies, are embedded in the static spacetime of a much larger vacuole (see, e. g. [8]), usually the vacuole of a larger structure, which is usually identified as the surrounding galaxy cluster. However, inside a vacuole there is no cosmological expansion, which would destroy the perhaps best established and most trusted cosmological instrument, Hubbles redshift distance relation. In other words, if gravitationally bound systems are embedded in a vacuole of the Einstein-Straus type, then most cosmological objects should, in first approximation, not be redshifted at all.

A more recent attempt to solve this problem is the inclusion of the cosmological constant Λ [9 – 11], an additional cosmological parameter connected to the vacuum energy, which was invented and then rejected by Einstein and which resurfaced only on account of more recent observations [12 – 14]. The cosmological constant reduces the Einstein-Straus radius by a factor of about 2 [5], which is by far not enough to abolish the mentioned underlying problem.

1.2. Local Spacetime and the PIONEER Anomaly

In this paper we let ourselves be challenged by the PIONEER anomaly (or PIONEER effect, subsequently abbreviated by PIO), which has been perceived as one of the most important unsolved problems in modern physics (see e. g. [15]). For at least two decades it is known that deep spaceprobes (most prominently the PIO-10 and -11 missions) seem to experience an anomalous deceleration towards the sun [16, 17]. While other spaceprobes, such as Galileo and Ulysses [17], also seem to observe this effect, the PIO spacecrafts are nevertheless the most appropriate ones for dynamical trajectory studies, since their spin stabilization and their great distances to the earth require only a minimum of artificial earth-attitude reorientations. This permits very precise Newtonian acceleration estimates down to the level of 10^{-10} cm/s^2. Since about 1980, when PIO-10 moved at solar distances larger than 20 AU (astronomical units) and the Newtonian solar gravitational pull dropped to levels of $a_{\mathrm{s}} \leq 5 \cdot 10^{-8}$ cm/s^2, the NASA Jet Propulsion Lab (JPL) orbit determination program (ODP) found unmodelled accelerations with a systematic residual level of $a_{\mathrm{PIO}} \simeq -(8.74 \pm 1.33) \cdot 10^{-8}$ cm/s^2 directed towards the sun. Interestingly enough, the level of these residual decelerations, besides some 10 percent fluctuations, remained constant for all the ongoing PIO itineraries to larger distances, i. e. seemed to prove as being independent of the solar distance, orientation and time.

A large number of proposals, how these anomalous decelerations could be explained, have meanwhile been made, see e. g. [16, 17] or [18]. These proposals mainly focus on classical and conservative physical effects like friction forces with interplanetary dust grains, asymmetric thermal emission from the probe or an accelerated motion of the whole solar system in the direction normal to the ecliptic. In addition, further deceleration effects were considered, such as the helicity-rotation coupling of the spacecraft [19, 20] or additional, general relativistic accelerations [21], although they all turned out to be systematically too small and/or position dependent to explain the observed effect. More speculative reasons were also discussed, like effects caused by dark matter or vacuum energy gravity contributions [11], but none of these proposed explanations up to now could fit the observed magnitude and the distance independence of the measured anomalous deceleration.

For this reason, more recently also purely cosmological causes for the existing anomalous deceleration have been added to the ongoing discussions, see e. g. [22 – 26]. These cosmological reasons of the PIO anomaly are all connected with the highly problematic question of how much the cosmological expansion of the universe may touch our local spacetime metrics near the sun [7, 11, 27], which in turn is closely related to the problem of the ES vacuole introduced in the first part of this section. One of the more promising ideas so far has been applied recently by Petry [24, 25], who seems to be able to explain the PIO anomaly in terms of an additional frequency shift, which occurs on account of the time dilatation during the propagation of the photon. His ansatz is built on a variant gravitational theory [28, 29], which, for weak gravitational fields, becomes identical with Einstein's general relativistic field equations [25]. We now adopt a modification of the standard cosmological representation which is trimmed to be "as close as possible" to the standard model. We critically analyze several possible modifications of this ES concept, attempting to reproduce a pseudo-cosmological frequency shift of radiophotons in a vacuole, which would be able to fix both problems, the redshift of distant photons and the observed blueshift of the PIO spacecrafts. This attempt is motivated by the observational fact that the PIO phenomenon, which is almost exclusively termed as anomalous deceleration, (which would turn out to be an illusion from this paper) is nicely represented by the cosmological quantity $a_{\rm PIO} \simeq H_0 c$, where H_0 denotes the present-day Hubble constant, which is of the order of 70 km/s/Mpc [14, 30].

2. Cosmological Redshift of Radiophotons

2.1. The Robertson-Walker Metric on a Local Scale

We begin our investigations by estimating what kind of redshift can be expected if full cosmological expansion happens on "local" scales such as that of the solar system. While the hypothesis of an omnipresent global expansion clearly contradicts the observational fact that distant galaxies are not systematically larger than close ones, we nevertheless feel it is a good "first step" in the general context of the problem which we attempt to explain, perhaps guiding the eyes to the really essential point to be considered here.

The phenomenon of the frequency shift registered at the radiolink to PIO-10 is well represented by the formula [17]

$$\Delta\nu_{\rm obs} = \nu_{\rm obs} - \nu_{\rm mod} = -\nu_0 \frac{2a_{\rm PIO} t_i}{c}, \tag{2}$$

where $2t_i$ is the time required for the radiophotons to travel from the earth to the probe and back, and $\Delta\nu$ has been normalized in a different way than it is usually done (see [17], ref. 38).

Repeating our calculations from a recent paper [26], the cosmological redshift in wavelength may be evaluated by applying the well-known relation

$$\frac{\lambda_i}{\lambda_0} = \frac{\nu_0}{\nu_i} = \frac{R_i}{R_0}, \tag{3}$$

where the quantities $\lambda_{0,i}$, $\nu_{0,i}$ and $R_{0,i}$ denote wavelength and frequency of the radiophoton and the scale of the universe at the times $t = 0$ and $t = t_i$, respectively. For $\Delta\nu$, this relation then leads to the formula

$$\Delta\nu_i = [\nu_0 - \nu_i] = \nu_0 \left[1 - \frac{R_0}{R_i}\right] = \nu_0 \left[1 - \frac{R_0}{R_0 + \dot{R}_0 t_i}\right], \tag{4}$$

which yields

$$\Delta\nu_i = \nu_0 \left[1 - \frac{1}{1 + H_0 t_i}\right] \simeq \nu_0 [H_0 t_i]. \tag{5}$$

Comparing this result with the PIO frequency shift (2), we derive the relation

$$\Delta\nu = -\nu_0 \frac{a_{\rm PIO} t_i}{c} = \nu_0 H_0 t_i. \tag{6}$$

This finally leads to the time-, distance- and direction-independent result

$$a_{\rm PIO} = -H_0 c, \tag{7}$$

which has, except for the wrong sign, exactly the observed order of magnitude and the desired orientation independence. This result suggests that a cosmologically inspired correction to the ES vacuole concept may, in fact, be able to explain the PIO anomaly.

2.2. The Modified Schwarzschild Vacuole

A problem which has often been recognized and theoretically studied is the question how local systems of gravitationally bound matter, like stars, galaxies and clusters of galaxies, might be embedded in the global, cosmological Robertson-Walker spacetime of the expanding universe. The problem of a needed connection between a central Schwarzschild metric and a time dependent outer Robertson-Walker metric had first been spotted in [1], and after that has been reinvestigated and revisited by many other authors [3, 4, 31 – 34]. As it turns out from all these studies, the exact connection between a time independent inner Schwarzschild metric of a non-rotating central mass M and a time dependent outer Robertson-Walker metric is only possible at the surface of a spherical vacuole, the often so-called Einstein-Straus vacuole, with a critical radius $r_{\rm ES}$ given by (1). This solution, however, leads to several problems, as already mentioned.

In a (static) ES vacuole there is absolutely no cosmological expansion term and absolutely no cosmological redshift. If these vacuoles truly exist and function as widely accepted, then they will fill significant parts of the universe, leading to considerable problems with the interpretation of the redshift of distant objects. So, strictly speaking, the ES solution seems to conflict with standard cosmology. Since the ES metric is equivalent to the Schwarzschild one [1], we begin our next approach to this problem by introducing an additional time dependence of the (inner) Schwarzschild metric. The only free parameter where we could introduce such a time dependence without destroying the spherical symmetry is the central mass $M = M(t)$, leading to a metric somewhat similar to the one developed in [32]. While this modification transforms the static Schwarzschild metric in a non-static one, strictly speaking this absolute static feature is already violated by the introduction of cosmologically time dependent boundary conditions in the Einstein-Straus problem [1]. In addition, and rigorously taken, the mere presence of additional, small masses around the central mass (e. g. planets) introduces small corrections to the metric, destroying both the spherical symmetry and the time independence of the solution.

Another argument allowing for the introduction of a time dependent central mass is the consideration that the requirement of a divergence-free energy-momentum-tensor,

$$T^{ab}_{;b} = 0, \tag{8}$$

only needs to be valid for the sum of all contributions to this tensor, and not necessarily for the individual contributions to this tensor only. Considering that, according to recent measurements [14, 30], most of the energy in the universe has a completely unknown form (dubbed dark energy or vacuum energy), it is highly improbable that there is no interaction between conventional (baryonic) matter and this unknown type of energy. We will come back to this point in Section 2.3.

Readers hesitating to follow such an idea may have a look into independent literature where time dependence of cosmic masses has been motivated as a viable concept and has been discussed in more details [32, 35 – 39]. In addition, in the recent papers [40] and [41] it has been shown that a cosmological increase of comoving cosmic masses would allow to fulfill both the equivalence principle of rotations in the universe and Mach‘s principle of related inertial masses.

For a photon moving out radially from a distance r_1 to r_2 in the time dependent Schwarzschild metric the following light geodetic relation is valid (for the elements of the Schwarzschild metric see e. g. the textbook by Goenner [42]):

$$r_2 - r_1 = c\int_{t_1}^{t_2} \frac{\mathrm{d}t}{1-\frac{2GM(t)}{c^2 r(t)}}. \tag{9}$$

Considering two photons which are sent within a time increment δt_1 from r_1 to r_2 and reflected back to r_1, one would then find the following time intervall δt_2 at the place r_1:

$$0 = c\delta t_2 \frac{1}{1-\frac{2GM_2}{r_1 c^2}} - c\delta t_1 \frac{1}{1-\frac{2GM_1}{r_1 c^2}}, \tag{10}$$

where $M_{1,2} = M(t_{1,2})$ has been introduced. From the above relation one further finds that

$$\frac{c\delta t_2}{c\delta t_1} = \frac{1-\frac{2GM_2}{r_1 c^2}}{1-\frac{2GM_1}{r_1 c^2}}, \tag{11}$$

and, with the obvious relation $1 \gg \frac{2GM(t)}{1-r(t)c^2}$, finally

$$\begin{aligned}\frac{c\delta t_2}{c\delta t_1} &\simeq \left(1-\frac{2GM_2}{r_1 c^2}\right)\left(1+\frac{2GM_1}{r_1 c^2}\right)\\ &\simeq 1-\frac{2GM_2}{r_1 c^2}+\frac{2GM_1}{r_1 c^2}.\end{aligned} \tag{12}$$

Identifying δt_1 as the time increment between the begin and end of a radio wavetrain, one thus can interprete the above relation also by the corresponding redshift formula

$$\frac{c\delta t_2}{c\delta t_1} = \frac{\lambda_2}{\lambda_1} \simeq 1-\frac{2GM_2}{r_1 c^2}+\frac{2GM_1}{r_1 c^2}. \tag{13}$$

All that remains now is to estimate the time dependence of the central mass, for which we make the ansatz $M_2 = M_1 + \dot{M}_1(t_2 - t_1)$. Then, from the above relation the typically introduced redshift $z = (\lambda_2 - \lambda_1)/\lambda_1$ is obtained by

$$\begin{aligned}z_{1,2} &= \frac{(\lambda_2-\lambda_1)}{\lambda_1} \simeq \frac{2G}{r_1 c^2}(M_1 - M_1 - \dot{M}_1(t_2-t_1))\\ &= -\frac{2GM_1}{r_1 c^2}\frac{\dot{M}_1}{M_1}(t_2-t_1).\end{aligned} \tag{14}$$

Comparing and identifying this result with the one obtained for the full cosmological redshift occuring during the passage time $(t_2 - t_1)$ (see Section 2.1), we arrive at the relation

$$\begin{aligned}\frac{\dot{R}_0}{R_0}(t_2-t_1) &= H_0(t_2-t_1)\\ &= -\frac{2GM_1}{r_1 c^2}\frac{\dot{M}_1}{M_1}(t_2-t_1),\end{aligned} \tag{15}$$

which obviously requires that

$$\frac{\dot{M}_1}{M_1} = -\frac{H_0}{\frac{2GM_1}{r_1 c^2}}. \tag{16}$$

When going to the black hole limit with $r_1 \rightarrow r_\mathrm{s} = 2GM_1/c^2$, then we would finally find

$$\frac{\dot{M}_1}{M_1} = -H_0. \tag{17}$$

This expresses the fact that the full cosmological redshift should also be seen in a Schwarzschild vacuole, whatever its size may be, if the central mass M decreases with time according to the above relation.

On the other hand, perhaps a more important point suggests itself: When looking at the result obtained with PIO, where one sees a blueshift instead of a redshift of this order of magnitude, then an evident solution would be to assume, instead of a decrease, just the opposite, namely an increase of the central mass according to

$$\frac{\dot{M}_1}{M_1} = H_0. \tag{18}$$

The semantic and physical interpretation of such a claim is of course not straightforward, but one statement at least can be made already now: The PIO enigma may be solved by assuming that the sun, and most probably also every other central star, is characterized by a circumsolar, or circumstellar, space with a Schwarzschild metric given for a time dependent central mass, varying with time according to the above relation. Hereby the central star is considered to be non-rotating and without a peculiar motion relative to the comoving cosmological rest frame. We discuss the possible impact of this result on cosmology (and other aspects of physics) in Sections 2.3 and 3.

2.3. Growing Masses and Dark Energy

We now present a possible explaination on how growing central masses might be interpreted in the current cosmological model of the universe. It is widely accepted that most parts of the universe are filled by a mysterious "dark energy", of which the true form still remains a complete mystery. We now estimate roughly a possible interaction between this dark energy and a time dependent Schwarzschild vacuole.

First, we may assume that the dark energy exerts pressure on the vacuole, and does work at its expansion [43]. If we assume that, by some unknown physical mechanism, this energy is converted into mass, we may approximate the resulting mass change by

$$\dot{M}c^2 = -(4\pi r_{\mathrm{ES}}^2)\dot{r}_{\mathrm{ES}}p_{\mathrm{v}}, \tag{19}$$

where p_{v} is the vacuum pressure. Representing this pressure by $p_{\mathrm{v}} = -\rho_{\mathrm{v}}c^2$, and the matter contained in the vacuole by $M = \frac{4\pi}{3}\rho_{\mathrm{m}}r_{\mathrm{ES}}^3$, then we may derive the following relation for the central mass gain:

$$\frac{\dot{M}}{M} = 3\left(\frac{\rho_{\mathrm{v}}}{\rho_{\mathrm{m}}}\right)\left(\frac{\dot{r}_{\mathrm{ES}}}{r_{\mathrm{ES}}}\right) \simeq 3\left(\frac{\rho_{\mathrm{v}}}{\rho_{\mathrm{m}}}\right)H_0. \tag{20}$$

If we consider that the density fraction is approximately $7/3$, according to WMAP results [14, 30], we obtain a growth rate of about $7H_0$, which agrees quite well with the ad hoc requirement assumed in the last section.

We would like to point out that this physical interpretation is only one of a large number of alternative possible ideas. Considering the huge number of aspects which are touched by our idea, it is simply not possible to derive a fully-fledged, conflict-free model in a single paper. The only thing that perhaps can be stated is that our current results are encouraging.

In addition, we would like to point out that a connection of vacuum energy and matter creation has already been discussed in the early papers [44 – 46], and the later ones [47 – 49]. For instance, the requirement that general relativistic field equations should be conformally invariant with respect to local scale recalibrations leads these authors to the introduction of a general relativistic action potential which describes mass generation connected with geodetic motions of particles. To describe this form of mass generation, a so-called C-field (creation-field) can be introduced which turns out to be connected with geodetic mass motions themselves. It then can be shown [49] that this C-field, when introduced into the general relativistic field equations, leads to terms equivalent to those resulting from vacuum energy. A similar connection between vacuum energy density and mass density was also found in [50], where it is shown that the cosmological term Λ should be proportional to the mass density, meaning that, when the latter is decreasing, the former should also decrease. Concluding these considerations, one can say that up to now there is surely a lack of a rigorous formulation for the transition of vacuum fluctuations into real masses. The most rigorous treatment so far of such a mechanism has been discussed in [51], where vacuum fluctuations in terms of particle-antiparticle production in the immediate neighbourhood of black holes should lead to the appearance of real particles. In this respect, the so-called Hawking radiation is a materialization of vacuum energy in a strong gravitational field. Perhaps, in this respect, the expanding universe also represents a form of a time dependent gravitational field including matter creation through the embedded time dependent quantum mechanical wavefunctions of particles.

2.4. The Modified Einstein-Straus Vacuole

We now advance to the Einstein-Straus vacuole, investigating what our modification from the previous section implies for the vacuole metric. The authors of [1] have proven that their solution is equivalent to a Schwarzschild metric, where the central mass M is constant. For this reason the solution which we have derived for the Schwarzschild metric may be transfered without any complication to the Einstein-Straus metric, by simply setting $M = M_0(1+H_0t)$. Then we obtain for the Einstein-Straus-radius

$$r_{\mathrm{ES}}(t) = \left(\frac{3M(t)}{4\pi\rho(t)}\right)^{1/3}, \tag{21}$$

which may be rewritten in terms of short timescales as

$$\begin{aligned} r_{\mathrm{ES}}(t) &\simeq \left(\frac{3M_0}{4\pi\rho_0}\frac{1\pm H_0t}{1-nH_0t}\right)^{1/3} \\ &\simeq \left(\frac{3M_0}{4\pi\rho_0}\right)^{1/3}\cdot(1+(n\pm 1)H_0t)^{1/3}. \end{aligned} \tag{22}$$

The parameter n is the power in the scale law

$$\rho(t) = \rho_0 a^{-n}(t), \tag{23}$$

where in standard cosmology it is usually assumed that $n = 3$. However, we would like to note that this value is not without any doubt, there being some hints that the cosmological matter density might scale differently e. g. the authors of [38, 39] obtained $n = 2$ from simple physical arguments. For $n > 1$, this result always leads to a growing vacuole radius; smaller values for n are highly unlikely. No matter which sign we select, this relation still predicts growing ES radii, which increases the problem of overlaping vacuoles. Since the explicit functional dependence of $\rho(t)$ itself is not absolutely known, we will not investigate this aspect in more detail in this paper.

3. Consequences of Stellar Mass Increase

3.1. The Schücking Relation

Schücking [3] has demonstrated that the mass removed from expanding space by the presence of an ES vacuole can be represented by

$$M = \frac{4\pi}{3}\rho_0 r_{\mathrm{ES}}^3. \tag{24}$$

Using the additional assumption that the only connection of the ES vacuole to the surrounding, expanding Robertson-Walker universe is the (implicit) time dependence of the ES radius, which happens because of the decreasing density $\rho_0 + \dot{\rho}_0\Delta t$, we obtain

$$\begin{aligned} \dot{M} &= \frac{4\pi}{3}[\dot{\rho}_0 R_{\mathrm{ES}}^3 + 3\rho_0 R_{\mathrm{ES}}^2\dot{R}_{\mathrm{ES}}] \\ &= \frac{4\pi}{3}\rho_0 R_{\mathrm{ES}}^3[\dot{\rho}_0/\rho_0 + 3\dot{R}_{\mathrm{ES}}/R_{\mathrm{ES}}], \end{aligned} \tag{25}$$

which simply yields the following relation:

$$\dot{M}/M = [\dot{\rho}_0/\rho_0 + 3H_0], \tag{26}$$

which, using (18), leads to the condition

$$\dot{\rho}_0/\rho_0 = -2H_0. \tag{27}$$

Interestingly enough, as can easily be confirmed, the above relation is fulfilled if ρ_0 scales with R_0^{-2}. Exactly this relation has, however, been derived as a requirement for a minimum and constant energy universe in [38] or [39]. So maybe PIO is just a hint for the fact that we are living in an economic universe with vanishing and constant total energy.

3.2. Newtonian Motions

We now estimate whether our modification to the ES metric does lead to conflicts with conventional celestial mechanics. We begin by investigating the two-body problem in a gravitationally bound system undergoing Keplerian motions. It is well known that the equations of motion in a central (Newton) potential are determined by the numerical value of the effective potential V_{eff}. This potential is proportional to the mass m of the moving object, which we assume to be constant in this first approximation; only the Newton potential depends on the central mass $M(t)$ as well. Thus the time dependent mass leads to the potential term

$$V(r) \propto \frac{M(t)m}{r} \simeq \frac{M_0 m}{r}(1\pm H_0t), \tag{28}$$

where, independent of the value of the central mass, we obtain a correction term of

$$H_0t \simeq o(10^{-11})\cdot\left(\frac{t}{1\text{ year}}\right) \ll 1, \tag{29}$$

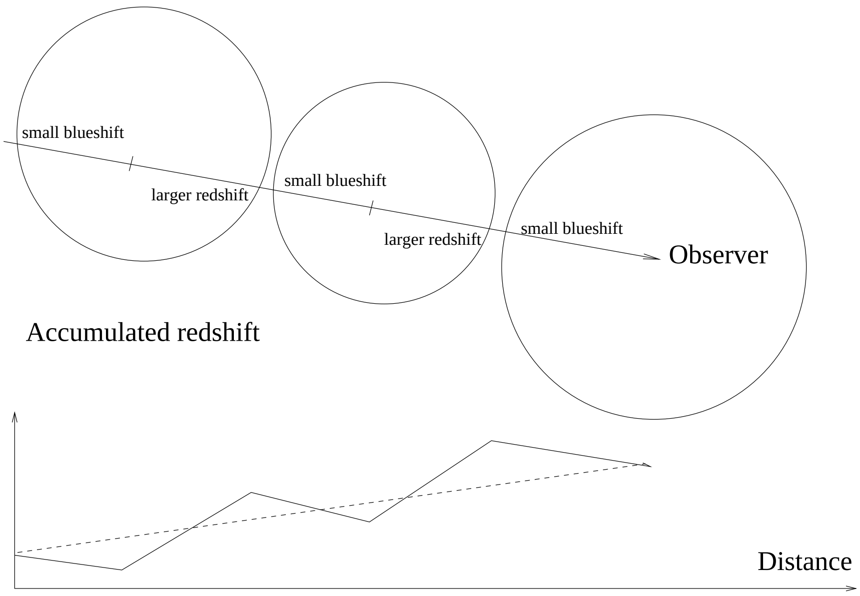

Fig. 1. Sketch on how the cosmological redshift may be reproduced in our ansatz. Since the central masses are growing, the outgoing effect (the redshift) is stronger than the ingoing effect (the blueshift). The vacuoles (circles) are assumed to be not overlapping.

which is so small that it can safely be considered unobservable, especially considering that other contributions (e. g. comets and dust accretion) also modify the mass of the contributing objects. Using the same argument, it is obvious that the mass m of the planet, if it undergoes a similar mass increase or decrease, does not lead to observable corrections, similar to all imaginable force terms where the mass is included in a rational function.

From this result we conclude that celestial mechanics is not affected in an observable way by our modifications, which is in excellent agreement with the arguments in [52], and also with a similar result in [53].

3.3. The Cosmological Redshift

A considerably more difficult problem related to our concept is the observationally well established redshift of distant astronomical objects, which, at first glance, seems to require a shrinking mass, instead of the growing mass used to explain the PIO anomaly. Since the fundamental problem with the ES vacuole, the total absence of redshifts, is not removed by a modified vacuole radius derived for a positive vacuum energy (see [11]), but instead is enhanced by vacuum energy contributions, our solution seems to be systematically conflicting with practically all astronomical observations (see Section 2.4).

We now demonstrate that this expectation is in fact wrong, and that the local blueshift is, at least qualitatively, compatible with the global cosmological redshift. If we assume that a photon emitted by a distant object towards the observer crosses a huge number of vacuoles on arbitrary trajectories (see Fig. 1), we may approximate the impact of an isolated vacuole by an effective redshift, which is composed of a blueshift gained on the path where the photon is getting closer to the central mass, and a redshift gained while the photon is leaving the vacuole. Since the mass is growing in time, the energy loss during the second half of the path is larger than the energy gain during the first half, and hence the photon is systematically redshifted. If, additionally, we assume that the photon did cross a huge number of vacuoles, then the contribution of the observer's vacuole, which the registered photon never crosses completely, may be safely ignored.

For the PIO anomaly, the effect is exactly the opposite: since the photon is outgoing (i. e. losing energy) first, and incoming (i. e. gaining energy) later, the returning photon will be bluer than the outgoing one, which is exactly what has been observed.

3.4. WMAP, the Cosmic Microwave Background and Dark Energy

Very recently, the WMAP experiment [14, 30] measured the cosmic microwave background (CMB), fitting cosmological parameters with unsurpassed precision. These results strongly favour the presence of a presently accelerated expansion of the universe, combined with a substantial amount of dark energy (Λ). This result does not, qualitatively, conflict with our representation, since the CMB is usually associated with a specific event in the very distant past, the decoupling of radiation after the big bang. However, the cosmological expansion has completely been left out from this paper, since we are not able to tell anything about it without making further assumptions. The presence of a unique event from which the CMB is derived is perfectly possible within our ideas, since the expansion of the vacuoles on account of the combined mass growth and density decrease (22) may still lead to a certain collective expansion effect, especially considering that the nature of dark energy is still a complete mystery. Thus, claiming that our ideas (which might as well lead to a variant cosmology, or not) contradict the big bang or the concept of cosmological expansion, without any further, more detailed investigation, would be premature at this point.

3.5. Experimental Verifications of our Model

Our results are based on one highly speculative assumption, namely that the local spacetime geometry acts as if it could be presented by a Schwarzschild metric with an increasing central mass. To test the viability of this assumption we propose to continue probing local spacetime by spaceprobes with a PIO-like radio-equipment.

First of all, to support our hypothesis, it has to be excluded that the so-called PIO anomaly is caused by an anomalous acceleration. This can relatively easily be carried out by not only monitoring the frequency shifts between outgoing and ingoing radiosignals, but by measuring, in addition, the signal transit time as function of the running time. In case of a regular Newtonian acceleration and a parabolic motion of the probe, light travel times $\tau = (R(t) - r_{\mathrm{E}})/c$ should simply increase with time t according to the relation given by the well-known Barker equation

$$6\frac{\mu^2}{C^3}(t-t_{\mathrm{p}}) = \Theta^3 + 3\Theta, \tag{30}$$

which also can be inverted to yield the relation [54]

$$\Theta(t) = \frac{6\Lambda}{(3\Lambda+\sqrt{1+9\Lambda^2})^2+1+(3\Lambda+\sqrt{1+9\Lambda^2})^{-2}}, \tag{31}$$

where the following notation was used:

$$\Lambda = \frac{\mu^2}{C^3}(t-t_{\mathrm{p}}). \tag{32}$$

Here t and t_{p} are the running time and the time of perihelion passage, respectively. The function $\Theta = \tan(\phi/2)$ is a function of the true anomaly of the probe, i. e. the angle ϕ covered after the perihelion passage, C denotes the angular momentum, and μ is defined by $\mu = GM_{\odot}$, with $M_{\odot}$ being the solar mass.

For the parabola the transit time τ is then found with the help of the well-known orbit equation

$$r(t) = \frac{p}{1+\cos\phi(t)}. \tag{33}$$

If it can be confirmed, while the probe is moving out to larger and larger distances, that transit times in fact behave like the Keplerian relation $\tau(t) = (r(\phi(t)) - r_{\mathrm{E}})/c$ requires, then it is proven that no anomalous acceleration acts.

For the case that no anomalous acceleration can be confirmed one has to ascribe the occuring radio frequency shift to the photon propagation itself, such as the action of time dependent spacetime itself on the free radiowave propagation. To test the special case of the shift derived in this paper, which is strongly dependent on the receiver position r_1, we suggest to use two probes communicating both with a receiver on earth and with each other, since then the dependence of $\Delta\nu$ on r could best be tested. In addition, this method also allows to exclude a possible dispersion effect of the radiophotons propagating through the local interplanetary medium.

3.6. Open Ends

As it happens with all new ideas, there is a fair amount of lose ends, which we have not investigated yet, mainly on account of the sheer complexity of the situation or of the great care that must be taken when trying to insert our ansatz more firmly into the existing foundations of physics and astronomy. We expect to evolve on these ideas in the future.

Among the problems which we definitely intend to revisit are a quantitative analysis of the cosmological redshift of distant objects and the question if our ansatz is truly compatible with the cosmological expansion and with dark energy. Only when we have cemented that the accepted concepts of standard cosmology are not invalidated by our ansatz, more dramatic consequences (i. e. "new physics") which might result from time dependent masses will also be investigated further.

However, we feel inclined to point out that our predictions may be easily verified (or disproved) by a simple, PIO-like experiment, where another spin-stabilized spacecraft equipped with nothing more than a radio receiver and emitter is thrown into the interplanetary medium, where special care is taken to measure the signal runtime, which is then compared with the predicted value. As we have already mentioned in Section 3.5, if the runtime points to a systematic deceleration, then our ansatz is proven invalid. If, however, a true acceleration can be clearly ruled out, then the observed frequency shifts must be an effect acting on the radiophotons themselves, such as the mechanism which we have proposed here. Such an experiment should be considerably less expensive and resource-eating than a full acceleration detector hunched on a satellite, such as the experiment proposed in [55].

4. Conclusions and Outlook

Faced with two existing conflicts in astronomy and cosmology, the PIO anomaly [15] and the missing redshift in a seemingly Einstein-Straus vacuole-dominated universe, we have investigated a modification of existing models under the main aspect of finding a solution to these two problems. We have managed to derive a possible solution for both of these seemingly unrelated problems, which consists of a modified Schwarzschild/Einstein-Straus metric, where the mass is not constant, but a function of time, scaling roughly similar to the cosmological scale factor R (i. e. $M = M_0(1 + H_0 t)$). Attempting to connect this ad hoc mass growth to a physical process, we have discovered that a possible interaction between conventional matter and dark energy is able to explain, phenomenologically, the modifications in the central masses.

We have also qualitatively investigated the conclusions which may be drawn for established astrophysics, namely the Schücking relation, celestial mechanics of the solar system and the redshift of distant objects; we were unable to find an obvious contradiction. Although these estimations definitely need to be repeated quantitatively, our initial results are nontheless very promising in attempting to explain a recognized problem (the PIO anomaly) and another, somehow less outstanding one (the Einstein-Straus redshift conflict).

[1] A. Einstein and E. G. Straus, Rev. Mod. Phys. **17**, 120 (1945).
[2] A. Einstein and E. G. Straus, Rev. Mod. Phys. **18**, 148 (1946).
[3] E. Schücking, Z. Phys. **137**, 595 (1954).
[4] W. B. Bonnor and P. A. Vickers, Gen. Rel. Grav. **13**, 29 (1981).
[5] R. Plaga, A&A **440**, L41 (2005).
[6] R. Plaga, Sterne und Weltraum **7**, 21 (2006).
[7] F. I. Cooperstock, V. Faraoni, and D. N. Vollick, Astrophys. J. **503**, 61 (1998).
[8] W. B. Bonnor, Astrophys. J. **316**, 49 (1987).
[9] R. Balbinot, R. Bergamini, and A. Comastri, Phys. Rev. D **38**, 2415 (1988).
[10] W. B. Bonnor, Class. Quant. Grav. **17**, 2739 (2000).
[11] A. D. Chernin, P. Teerikorpi, and Y. V. Baryshev, A&A **456**, 13 (2006).
[12] S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knoop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, D. E. Groom, I. M. Hook, A. G. Kim, M. Y. Kim, J. C. Lee, N. J. Nunes, R. Pain, C. R. Pennypacker, and R. Quimby, Astrophys. J. **517**, 565 (1999).
[13] C.J. Hogan, R.P. Kirshner, and N.B. Suntzeff, Sci. Am. **280**, 28 (1999).
[14] D.N. Spergel, L. Verde, H.V. Peiris, E. Komatsu, M. R. Nolta, C. L. Bennett, M. Halpern, G. Hinshaw, N. Jarosik, A. Kogut, M. Limon, S. S. Meyer, L. Page, G. S. Tucker, J.L. Weiland, E. Wollack, and E.L. Wright, Astrophys. J.S **148**, 175 (2003).
[15] M. Brooks, New Sci. **2491**, 30 (2005).
[16] J. D. Anderson, P. A. Laing, E. L. Lau, A. S. Liu, M. M. Nieto, and S. G. Turyshev, Phys. Rev. Lett. **81**, 2858 (1998).
[17] J. D. Anderson, P. A. Laing, E. L. Lau, A. S. Liu, M. M. Nieto, and S. G. Turyshev, Phys. Rev. D **65**, 082004 (2002), for an updated version see gr-qc/0104064.
[18] M. M. Nieto and S. G. Turyshev, Class. Quant. Grav. **21**, 4005 (2004).
[19] B. Mashhoon, Phys. Lett. A **306**, 66 (2002).
[20] J. D. Anderson and B. Mashhoon, Phys. Lett. A **315**, 199 (2003).
[21] M. Carrera and D. Giulini, On the influence of the global cosmological expansion on the local dynamics of the solar system (2006), gr-qc/0602098.
[22] J. L. Rosales and J. L. Sanchez-Gomez, The "Pioneer effect" as a manifestation of the cosmic expansion in the solar system (1999), gr-qc/9810085.
[23] J. L. Rosales, The Pioneer's Anomalous Doppler Drift as a Berry Phase (2004), gr-qc/0401014.
[24] W. Petry, Z. Naturforsch. **60a**, 255 (2005).

[25] W. Petry, in: Physical Interpretations of Relativity Theory X, Imperial College, London 2006, see also: arXiv: physics/0509173.
[26] H.-J. Fahr and M. Siewert, Does PIONEER measure local spacetime expansion? (2006), gr-qc/0610034.
[27] C. Lämmerzahl, O. Preuss, and H. Dittus, Is the physics within the solar system really understood? (2006), gr-qc/0604052.
[28] W. Petry, Gen. Rel. Grav. **13**, 865 (1981).
[29] W. Petry, Gen. Rel. Grav. **13**, 1057 (1981).
[30] C.L. Bennett, M. Halpern, G. Hinshaw, N. Jarosik, A. Kogut, M. Limon, S.S. Meyer, L. Page, D.N. Spergel, G.S. Tucker, E. Wollack, E.L. Wright, C. Barnes, M.R. Greason, R.S. Hill, E. Komatsu, M.R. Nolta, N. Odegard, H.V. Peiris, L. Verde, and J.L. Weiland, Astrophys. J. **148**, 1 (2003).
[31] G. Järnefelt, Z. Astrophys. **7**, 326 (1933).
[32] G.C. McVittie, Mon. Not. R. Astron. Soc. **93**, 325 (1933).
[33] C. Gilbert, Mon. Not. R. Astron. Soc. **116**, 678 (1956).
[34] W.B. Bonnor, Mon. Not. R. Astron. Soc. **282**, 1467 (1996).
[35] G.J. Whitrow, Nature **158**, 165 (1946).
[36] H. Hönl and H. Dehnen, Z. Astrophys. **68**, 181 (1968).
[37] J.M. Overduin and H.-J. Fahr, Naturwissenschaften **88**, 491 (2001).
[38] H.-J. Fahr, in: Knowledge and Belief – Wissen und Glauben, 26th Int. Wittgenstein Symposium (Eds. W. Loeffler and P. Weingaertner), öbvhpt Verlag, Wien 2004, pp. 339 – 353.
[39] H.-J. Fahr and M. Heyl, Astron. Nachr. **327**, 733 (2006).
[40] H.-J. Fahr, Found. Phys. Lett. **19**, 423 (2006).
[41] H.-J. Fahr and J. Zönnchen, Naturwissenschaften **93**, 577 (2006).
[42] H. Goenner, Einführung in die Kosmologie, Spektrum Akademischer Verlag, Heidelberg 1997.
[43] P.J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003).
[44] F. Hoyle, Mon. Not. R. Astron. Soc. **108**, 372 (1948).
[45] F. Hoyle and J.V. Narlikar, Proc. R. Soc. London A **290**, 143 (1966).
[46] F. Hoyle and J.V. Narlikar, Proc. R. Soc. London A **290**, 162 (1966).
[47] F. Hoyle, Astrophys. Space Sci. **168**, 59 (1990).
[48] F. Hoyle, Astrophys. Space Sci. **198**, 195 (1992).
[49] F. Hoyle, G. Burbidge, and J. V. Narlikar, Astrophys. J. **410**, 437 (1992).
[50] C. Massa, Astrophys. Space Sci. **215**, 59 (1994).
[51] S.W. Hawking, Comm. Math. Phys. **43**, 199 (1975).
[52] R.H. Dicke and P.J. E. Peebles, Phys. Rev. Lett. **12**, 435 (1964).
[53] L. Iorio and G. Giudice, New Astron. **11**, 600 (2006).
[54] W. Neutsch and K. Scherer, Celestial Mechanics: An Introduction to Classical and Contemporary Methods, B.I. Wissenschaftsverlag, Mannheim 1992.
[55] M.M. Nieto, S.G. Turyshev, and J.D. Anderson, The Pioneer Anomaly: The Data, its Meaning and a Future Test (2004), gr-qc/0411077.